Exercises for the lecture of Manuel Kauers held on 2011/12/16

Consider the holonomic squence $(a_n)_{n=0}^{\infty}$ defined via the recurrence

$$\begin{aligned}&(32858200 + 11197935n + 828539n^2)a_n\\&\quad - (83090130 + 40642646n + 4009948n^2)a_{n+1}\\&\qquad + (5170470 + 17680361n + 3181409n^2)a_{n+2} = 0\end{aligned}$$

and $a_0 = -18$, $a_1 = 80$.

1. Show that the sequence $(a_n)_{n=0}^{\infty}$ is not only holonomic, but even polynomial. Proceed as follows: First, use the recurrence to calculate some further terms of the sequence. Next, compute the interpolating polynomial of the terms you determined. Finally, prove that the polynomial you found indeed satisfies the recurrence.

   This part of the exercise is meant to be solved using computer algebra software. (A file with a Maple/Mathematica friendly version of the recurrence is posted on the course homepage. No need to retype it.)

2. Using the result obtained in part 1, compute a closed-form representation of $\sum_{k=0}^{n} a_k$ using one of the methods explained in the course.

   This part of the exercise is meant to be solved by paper and pencil.