

Homework for lecture by F.Winkler

“Fundamentals of Numerical Analysis and Symbolic Computation”, WS 2012/13

- (1) Consider the system of linear equations

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$$

Compute a Gröbner basis for this system of linear polynomials; you may use a computer algebra system like MATHEMATICA or MAPLE. If possible, show also the intermediate results. Interpret the output in terms of Gaussian elimination on the extended matrix of this linear system.

- (2) Consider the following polynomials in $\mathbb{Q}[x]$:

$$\begin{aligned} f &= x^6 - 2x^5 - x^4 - 4x^3 - 5x^2 - 2x - 3, \\ g &= 3x^6 - x^5 + 4x^4 - 2x^3 + 2x^2 - x + 1. \end{aligned}$$

Compute the greatest common divisor of f and g ; you may use a computer algebra system like MATHEMATICA or MAPLE. If possible, show also the intermediate results. Interpret the output in terms of the Euclidean algorithm.