FUNDAMENTALS

of Numerical Analysis and Symbolic Computation

- Exercises on Discontinous Galerkin Methods -

WS 2012/2013

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Exercise 1: Show that the gradient ∇u of the weak solution $u \in V_0 := H_0^1(\Omega)$ of the variational problem (1) belongs not only to $[L_2(\Omega)]^d$ but also to $H(\mathrm{div}) := \{v \in [L_2(\Omega)]^d : \mathrm{div}(v) \in L_2(\Omega)\}$, and, moreover, $\mathrm{div}(\nabla u) = -f$!

Exercise 2: Show the DG-identities (3), i.e.

$$\sum_{\delta \in \mathcal{T}_h} (\nabla u \cdot n, v)_{\partial\delta} = \sum_{\delta \in \mathcal{T}_h} (\{\nabla u\}, v \cdot n)_{\partial\delta} = \sum_{e \in \overline{\mathcal{E}}_h} (\{\nabla u\}, [v])_e$$

for a weak solution $u \in H_0^1(\Omega) \cap H^s(\mathcal{T}_h)$ of the variational problem (1) and for all $v \in H^s(\mathcal{T}_h)$ with some $s > 3/2$!

Exercise 3: Prove the consistency theorem: Let $s > 3/2$. Then the following statements are valid:

1. Assume that the weak solution u of (1), i.e. the solution of $(1)_{\mathrm{VF}}$ ($\exists$! due to Lax & Milgram), belongs to $H^s(\mathcal{T}_h)$. Then u satisfies the DG variational formulation (5).
2. Conversely, if $u \in H_0^1(\Omega) \cap H^s(\mathcal{T}_h)$ satisfies the DG variational formulation (5), then u is also the solution of our variational problem $(1)_{\mathrm{VF}}$.

Exercise 4: Show that the Dirichlet boundary condition $u = 0$ on Γ is incorporated in (5) resp. (6) ! Derive the DG variational formulation (5) resp. the DG scheme (6) for the case of inhomogeneous Dirichlet boundary conditions $u = g$ on Γ and piecewise constant coefficients $a|_\delta = a_\delta = \mathit{const} > 0$ for all $\delta \in \mathcal{T}_h$!.

Remark: All references (number) refer to formula markers from the lectures, see also lecture notes !