Example Group Theory: We consider the axioms

$$
\begin{aligned}
&(1) \quad 1 \cdot x = x, \\
&(2) \quad x^{-1} \cdot x = 1, \\
&(3) \quad (x \cdot y) \cdot z = x \cdot (y \cdot z).
\end{aligned}
$$

We orient the equations into rules and order the terms according to the lexicographic path ordering with the following preference of operators:

$$
{}^{-1} \ > \ \cdot \ > \ 1
$$

(compare Avenhaus Section 3.7).

This leads to the following rules generated by the KB completion algorithm:

(1)	$1 \cdot x \to x,$	
(2)	$x^{-1} \cdot x \to 1,$	
(3)	$(x \cdot y) \cdot z \to x \cdot (y \cdot z),$	
(4)	$x^{-1} \cdot (x \cdot y) \to y$	from 2 and 3,
(5)	$1^{-1} \cdot x \to x$	from 1 and 4,
(6)	$(x^{-1})^{-1} \cdot 1 \to x$	from 2 and 4,
(7)	$(x^{-1})^{-1} \cdot y \to x \cdot y$	from 6 and 3,
(8)	$x \cdot 1 \to x$	from 6 and 7,
(9)	$1^{-1} \to 1$	from 2 and 8,
(10)	$(x^{-1})^{-1} \to x$	from 7 and 8,
(11)	$x \cdot x^{-1} \to 1$	from 10 and 2,
(12)	$x \cdot (y \cdot (x \cdot y)^{-1}) \to 1$	from 3 and 11,
(13)	$x \cdot (x^{-1} \cdot y) \to y$	from 11 and 3,
(14)	$(x \cdot y)^{-1} \cdot (x \cdot (y \cdot z)) \to z$	from 3 and 4,
(15)	$x \cdot (y \cdot ((x \cdot y)^{-1} \cdot z)) \to z$	from 13 and 3,
(16)	$x \cdot (y \cdot (z \cdot (x \cdot (y \cdot z))^{-1})) \to 1$	from 12 and 3,
(17)	$x \cdot (y \cdot x)^{-1} \to y^{-1}$	from 12 and 4,
(18)	$x \cdot ((y \cdot x)^{-1} \cdot z) \to y^{-1} \cdot z$	from 17 and 3,
(19)	$x \cdot (y \cdot (z \cdot (x \cdot y))^{-1}) \to z$	from 17 and 3,
(20)	$(x \cdot y)^{-1} \to y^{-1} \cdot x^{-1}$	from 17 and 4.

Once a confluent and terminating rule system (1) – (20) has been computed, some of the rules in the system can be eliminated. Suppose that the left hand side of the rule (a) $l_a \to r_a$ can be reduced by the rule (b) $l_b \to r_b$. Then in computing normal forms for terms the rule (a) is superfluous, because whenever a term is reducible by (a) it is also reducible by (b). Finally, however, a unique normal form is reached. So the rule (a) can be deleted from the rule system without changing the corresponding equational theory and without destroying the confluence of the rule system.

Applying this elimination process to the rule system for group theory (1) – (20), we finally get the following confluent and terminating rule system:

$$
\begin{array}{ll}
(1) & 1 \cdot x \to x, \\
(2) & x^{-1} \cdot x \to 1, \\
(3) & (x \cdot y) \cdot z \to x \cdot (y \cdot z), \\
(4) & x^{-1} \cdot (x \cdot y) \to y, \\
(5) & x \cdot 1 \to x, \\
(6) & 1^{-1} \to 1, \\
(7) & (x^{-1})^{-1} \to x, \\
(8) & x \cdot x^{-1} \to 1, \\
(9) & x \cdot (x^{-1} \cdot y) \to y, \\
(10) & (x \cdot y)^{-1} \to y^{-1} \cdot x^{-1}.
\end{array}
$$