Let $M = 4x - 4Y^2 + 13xY^2 + 4Y^3 + 4xY^4 \in \mathbb{Q}(x)[Y]$ and let y be such that $M(y) = 0$.

Task 1 Compute $a_0, a_1, a_2, a_3 \in \mathbb{Q}(x)$ such that $\frac{d}{dx}y = a_0 + a_1 y + a_0 y^2 + a_3 y^3$.

Task 2 Compute a polynomial $M' \in \mathbb{Q}(x)[Y]$ such that $M'(y^2 + xy - 1) = 0$.

Task 3 Compute a polynomial $M'' \in \mathbb{Q}(x)[Y]$ with $M''\big(y \circ \sqrt{1 - x^2}\big) = 0$.

Task 4 Compute the first few terms of the four series expansions of y.

Task 5 Compute the singular points of y.

Task 6 Determine the asymptotic behaviour of the coefficient sequence $(a_n)_{n=0}^{\infty}$ in the series expansion $\sum_{n=0}^{\infty} a_n x^n$ of the branch of y going through the place $(0, 1)$.

You are welcome to do all the requested calculations using a computer algebra system, and to submit a transcript of your session as solution.