Home Work

for the Lecture of Bruno Buchberger
in the Frame of the Course

“Fundamentals
of Numerical Analysis and Symbolic Computation”

2016 06 01

Task 1

Give three examples of three polynomial equation systems in three unknowns, one example that has no solution, one example that has finitely many solutions, one example that has infinitely many solutions.

Use the Gröbner bases method (calling the Mathematica GroebnerBasis function) for arguing whether the systems have no, finitely many, or ifinitely many solutions. In the finite case, determine all the solutions starting from the Gröbner basis. In the infinite case, try to say something about the solution manifold. For solving univariate equations that may occur during this procedure, you may use the built-in Mathematica Solve function.

Task 2

Let’s represent the natural numbers by 0 and the successor function $^{+}$ i.e., in this representation, the natural numbers “are” 0, 0^{+}, 0^{++}, 0^{+++}, etc.

Define the function + inductively by:

$$\forall_{m} \quad (m + 0 = m)$$
$$\forall_{m,n} \quad (m + n^{+} = (m + n)^{+}) \,.$$

Prove that

$$\forall_{m,n} \quad (m + n = n + m) \,.$$

Here, the quantifier ∀ ranges over the natural numbers.