Reconstruction of ultra-short laser pulses

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Doctoral Program
Computational Mathematics
Numerical Analysis and Symbolic Computation

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- Physical Background
  1. Motivation
  2. SD-SPIDER method
  3. Equation
  4. Identifiability
- Mathematical Analysis
- Numerical treatment
  1. Discretization
  2. Regularization
  3. Choice of the starting phase
  4. Choice of the regularization parameter
- Results for simulated data
- Real data situation

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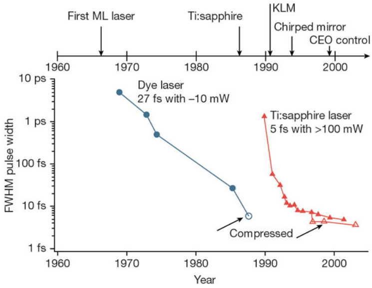
Why study ultra-short laser pulses?
to create shorter, stronger pulses; to enhance optical systems; medicine, material processing, etc.

Development of pulse durations:



Problem: measurements limited by electronics (order 10^{-12}s)
Solution: sample pulse by itself

Overview

- SD-SPIDER=
  Self-Defraction Spectral Phase Interferometry for Direct Electric-field Reconstruction
- introduced by the research group ‘Solid State Light Sources’ led by Dr. Günter Steinmeyer as subdivision of division C ‘Nonlinear Processes in Condensed Matter’ at Max-Born-Institute for Nonlinear Optics and Short Pulse Spectroscopy, Berlin
- theory presented at Conference on Lasers and Electro-Optics, 2010
- reasons for introduction: applicable for ultraviolet radiation, good signal strength

Basics of nonlinear optics:

Polarization $\tilde{P}$ caused by an electric field $\tilde{E}$,

$$\tilde{P}(t) = \epsilon_0[\chi^{(1)}\tilde{E}(t) + \chi^{(2)}\tilde{E}^2(t) + \chi^{(3)}\tilde{E}^3(t) + \dots] \qquad (1)$$

may act as source of electromagnetic radiation:

$$\nabla \times (\nabla \times E) + \frac{n^2}{c^2}\partial_t^2 E = -\mu_0 \partial_t^2 P_{\mathsf{NL}}(E) \qquad (2)$$

Refraction index n and Kerr-effect:

$$n(\omega) = n_0 + n_2|E(\omega)|^2, \qquad (3)$$

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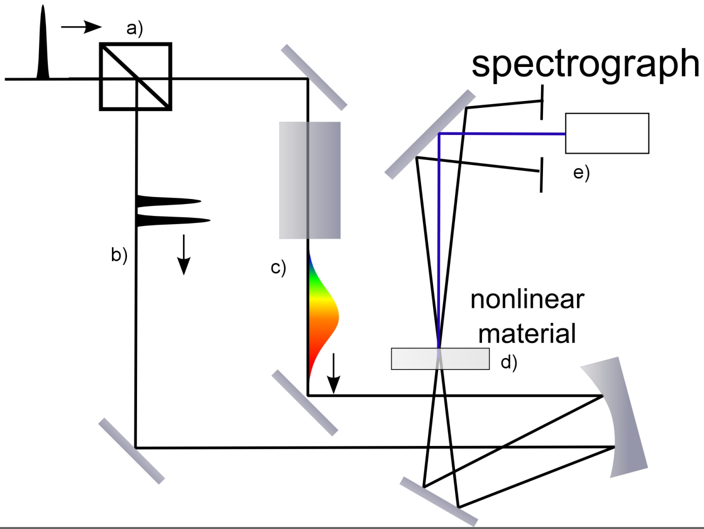
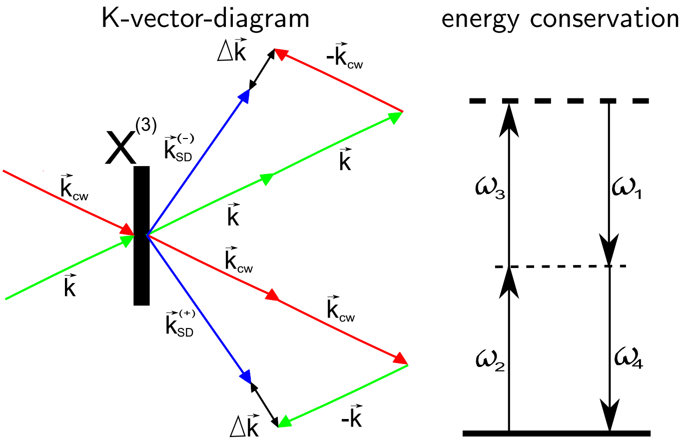
Principle:





Problem: measured signal is an autoconvolution of the fundamental pulse

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$$F[x](s) = \int_0^s k(s,t)x(t)x(s-t)dt = y(s) \tag{4}$$

$$Fx = y \qquad 0 \le t \le 1, 0 \le s \le 2 \tag{5}$$

continuous, complex valued kernel (in physical formulation)

$$K(\omega,\hat{\omega}) = \frac{\mu_0 c L}{2}\frac{\omega}{n(\omega)}\chi^{(3)}(\omega,-\omega_{cw},\hat{\omega},\omega+\omega_{cw}-\hat{\omega}) \overline{\mathcal{E}}^{cw} e^{i(\Delta\vec{k}_\xi \xi + \Delta\vec{k}_\eta \eta + \Delta\vec{k}_\zeta \frac{L}{2})} sinc(\Delta\vec{k}_\zeta \frac{L}{2}) \tag{6}$$

fundamental pulse: $x(t) = |x(t)|e^{i\varphi_x(t)} \in \mathbb{C}$

measured SD-pulse: $y(s) = |y(s)|e^{i\varphi_y(s)} \in \mathbb{C}$

to be reconstructed: $\varphi_x(t) = \varphi_0 + \int_{-\infty}^{t} GD(\tau)d\tau$

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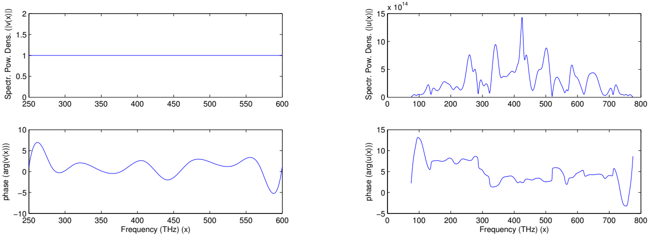
at first only φ_y and $|x|$ available
Does $|y|$ have to be measured too?

Yes.
Example with $k(s,t) \equiv 1$:



$\Rightarrow$ available data $|y|$, φ_y, $|x|$

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$$Fx = y, \qquad F : L^2[0,1] \mapsto L^2[0,2] \tag{7}$$

- $F(x)$ continuous
- Fréchet-derivative
  $$[F'(x_0)h](s) = \int_0^s (k(s,t) + k(s,s-t))x_0(s-t)h(t)dt$$
- $F(x)$ in general non-compact
- Fréchet-derivative always compact
- $F(x)$ everywhere locally ill-posed

Def.: We define an operator of type (7) to be locally ill-posed in x_0 if, for arbitrarily small $\rho > 0$ there exists a sequence $\{x_n\} \subset B_\rho(x_0)$ satisfying the condition

$$F(x_n) \to F(x_0) \text{ in } Y \text{ as } n \to \infty, \text{ but } x_n \nrightarrow x_0 \text{ in } X.$$

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Injectivity:

For $k(s,t) \equiv 1$ and $k(s,t) = k(s)$: $F(x_1) = F(x_2)$ has two solutions $x_1 = x_2$ and $x_1 = -x_2$ by Titchmarsh's theorem.

For $k(s,t)$ again $x_1 = x_2$ or $x_1 = -x_2$, additional solutions are an open problem.

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Equation: $y(s) = \int_0^s k(s,t)x(s-t)x(t)dt$
$supp(x) = [t_l, t_u]$, $supp(y) = [2t_l - t_{cw}, 2t_u - t_{cw}]$
discretization using rectangular rule

$$y(s_m) = \sum_{j=1}^{N} k(s_m, t_j)x(s_m + t_{cw} - t_j)x(t_j)\Delta t$$

with $\Delta t = \frac{t_u - t_l}{N-1}$, $t_j = t_l + (j-1)\Delta t$, $s_m = 2t_j + (m-1)\Delta t$
$y_m := y(s_m)$, $x_n := x(t_n)$, $k_{m,n} := k(s_m, x_n)$

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in matrix-form $\underline{y} = \underline{F(x)}\,\underline{x}$, with

$$\underline{y}/\Delta t = \underline{F}\,\underline{x}/\Delta t =$$

$$\begin{pmatrix}
k_{1,1}x_1 & 0 & \dots & 0 & 0 \\
k_{2,1}x_2 & k_{2,2}x_1 & \dots & 0 & 0 \\
 & \ddots & \ddots & & \vdots \\
k_{N-1,1}x_{N-1} & k_{N-1,2}x_{N-2} & \dots & k_{N-1,N-1}x_1 & 0 \\
k_{N,1}x_N & k_{N,2}x_{N-1} & \dots & k_{N,N-1}x_2 & k_{N,N}x_1 \\
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\end{pmatrix}
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Decomposition, with $\circ$ as element-by-element multiplication:

$\underline{F} = \underline{K} \circ \underline{X}$

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$$[F'(x_0)h](s) = \int_0^s (k(s,t) + k(s,s-t))x_0(s-t)h(t)dt$$

discretized with rectangular rule:

$$[\underline{F'(x_0)}\underline{h}]_m = \sum_{j=0}^{N} (k(s_m,t_j)+k(s_m,s_m+t_{cw}-t_j))x_0(s_m+t_{cw}-t_j)h(t_j)\Delta t,$$

resulting matrix $\underline{F'(x_0)} = (\underline{K} + \underline{K'}) \circ \underline{X_0}$

advantage: time-consuming calculation of the matrices $\underline{K}$ and $\underline{K'}$ has to be performed only once for each measurement setup

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Iterative linearized Tikhonov-type Regularization

$x_k =$
argmin$||y^\delta - F(x_{k-1}) - F'(x_{k-1})(x - x_{k-1})||^2 + \alpha_k ||L(x - x_{k-1})||^2$
with $Lz = z''$ as approximation of the second derivative

Iteration rule:

$$\underline{x}_{k+1} = \underline{x}_k + (\underline{F}'(\underline{x}_k)^* \underline{F}'(\underline{x}_k) + \alpha_k \underline{L}^* \underline{L})^{-1} \underline{F}'(\underline{x}_k)^* (\underline{y}^\delta - \underline{F}(\underline{x}_k)). \quad (8)$$

$\alpha_k = \alpha = const$

starting value $x_0 = |x^\delta| e^{(i\varphi_{\text{Start}})}$

iteration stops if $||\underline{y}^\delta - \underline{F}(\underline{x}_{k+1})||_2 \geq q ||\underline{y}^\delta - \underline{F}(\underline{x}_k)||_2$,
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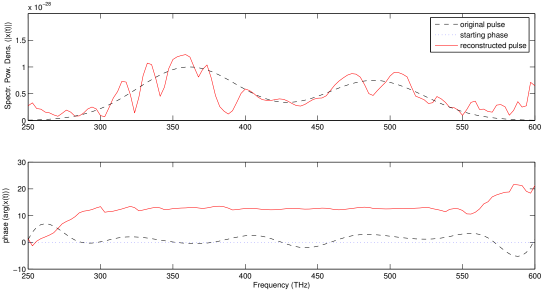
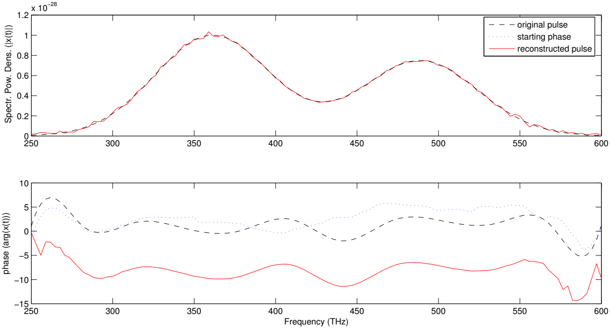
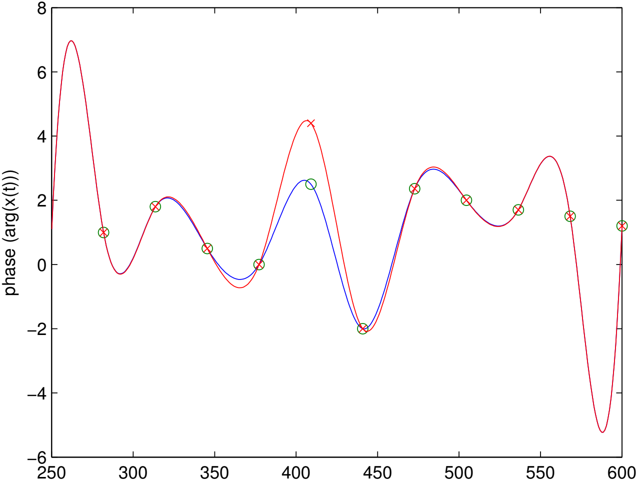
first idea: $\varphi_{\text{start}}(t) = 0$



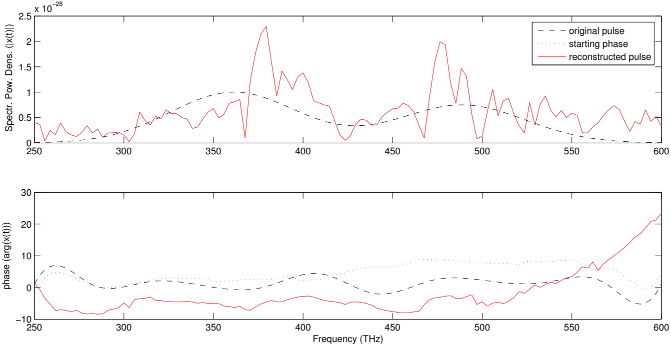
idea: $\varphi_{\text{start}}(t) = \frac{1}{2}(P(\varphi_y(s))) - \varphi_k(s^*, t)$
with kernel-phase $\varphi_k(s^*, t)$ for $s^* = 475THz$



problem for slightly changed fundamental phase

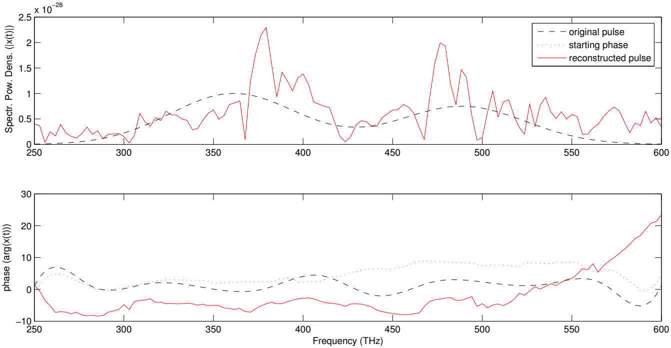


best result with kernel correction



⇒ set starting phase to constant zero

best result with kernel correction



⇒ set starting phase to constant zero

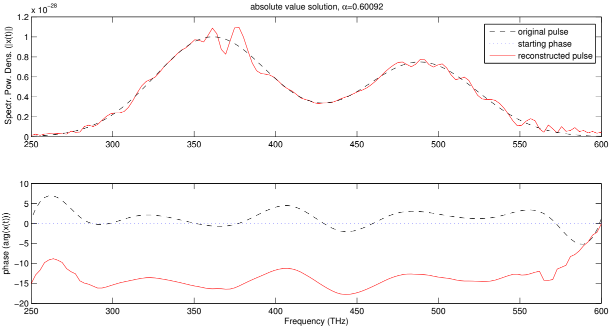
Overview

no a-priori information $||y - y^\delta|| < \delta$ available, thus a-posteriori methods necessary

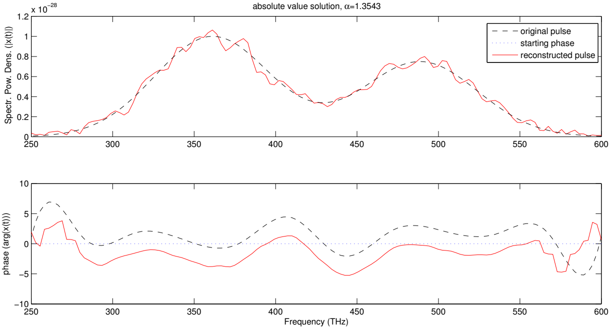
- L-curve not applicable
- quasioptimality ($||x_{\alpha_{i+1}} - x_{\alpha_i}|| \to min$) - failed
- absolute value method ($|||x^\delta| - |x^*_{\alpha_\ell}||| \to min$) - very reliable

Overview

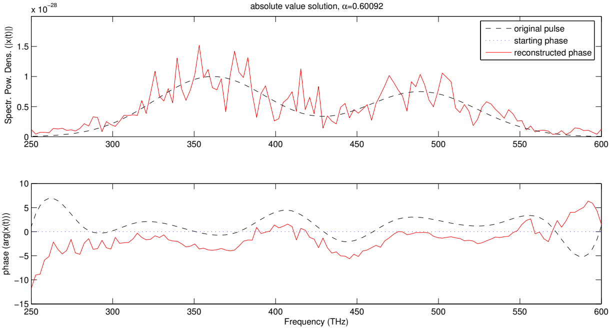
reconstruction for $\delta = 0.1\%$



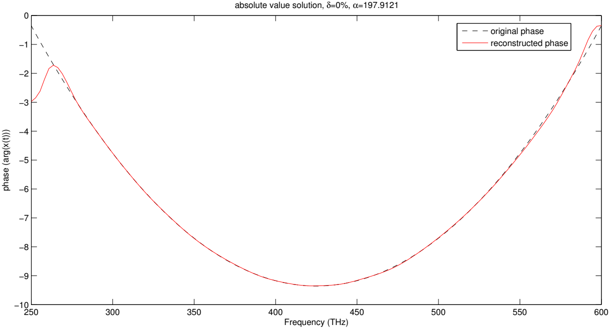
reconstruction for $\delta = 1\%$



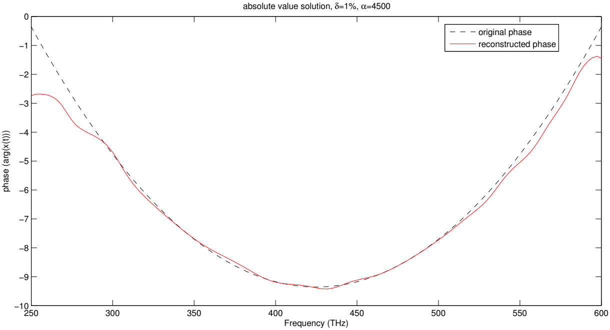
reconstruction for $\delta = 5\%$



reconstruction for $\delta = 0\%$



reconstruction for $\delta = 1\%$



Overview

unfortunately, no results available. Main reasons:

- measurements without magnitudes
- unknown factor in model $\Rightarrow$ error in the computational model
- frequency domains of x and y do not match

Thank you for your attention!