Integral Equations and Boundary Value Problems

Exercise, WS 2018/19

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13. Solve the following integral equation using the matrix approach:

$$g(s) - \frac{1}{\pi}\int_0^{2\pi} \sin(s+t)g(t)\,dt = 1\,, \qquad s \in [0, 2\pi]\,.$$

14. Please prove or contradict:

   (a) The space $C[-1,1]$ is complete wrt the L^2-norm.

   (b) The differential operator $D : C^1[0,1] \to C[0,1]$ is continuous, where the spaces $C^1[0,1]$ and $C[0,1]$ are equipped with the maximum-norm $\|\cdot\|_\infty$.

15. Solve the following integral equation using the Laplace transform:

$$\phi(x) = x + \int_0^x (x-y)\phi(y)\,dy\,.$$

Hint: Use that $\mathcal{L}(\int_0^x g(x-y)f(y)\,dy) = \mathcal{L}(g)\mathcal{L}(f)$ and that $(\mathcal{L}t)(s) = \frac{1}{s^2}$ and $(\mathcal{L}\sinh)(s) = \frac{1}{s^2-1}$.

16. Solve the following integral equation (find one special solution):

$$3\sin(x) + 2\cos(x) = \int_{-\pi}^{\pi} \sin(x+y)\phi(y)\,dy\,, \quad x \in [-\pi, \pi]\,.$$

17. Solve the following integral equation:

$$\phi(x) = x^2 + \lambda\int_0^1 x^3 s^2 \phi(s)\,ds\,, \quad \lambda \neq 6\,.$$

18. Find all continuous solutions of the integral equation

$$\int_a^b x(t)\,dt = g(s), \qquad s \in [a,b]\,,$$

in terms of the given function $g \in C[a,b]$.