Integral Equations and Boundary Value Problems

Exercise, WS 2018/19

Exercise sheet 9 12.12.2018

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49. Please solve:

$$\int_0^s (s-t)x(t)\,dt = \sin(s) - s\,, \qquad s \in [0,1]\,.$$

50. Let $K : L^2[0,1] \to L^2[0,1]$ be the integral operator induced by the kernel

$$k(s,t) := 4\pi^2 \begin{cases} (1-s)t\,, & t \leq s\,, \\ (1-t)s\,, & t > s\,. \end{cases}$$

Compute $\mathcal{N}(I-K)$ and $\mathcal{N}(I-K')$.

51. Let K be as in Exercise 50. Under which conditions exists a solution of the inhomogeneous equalities $x - Kx = f$ and $y - K'y = g$ in $L^2[0,1]$?

52. Let H be a Hilbert space and $T \in \mathcal{L}(H)$. Please show:

    (a) T invertible $\Longrightarrow$ T^{-1} commutes with each operator which commutes with T.

    (b) $K \in \mathcal{L}(H)$ compact and $\lambda I - K$ invertible $\Longrightarrow$ $(\lambda I - K)^{-1}$ can be represented as sum of a multiple of I and a compact Operator J.

53. Let H be a complex Hilbert space and $T \in \mathcal{L}(H)$ be *skew-adjoint*, i.e., $T^* = -T$. Show that $\sigma(T) \in \{ix : x \in \mathbb{R}\}$. Use this fact to show that if $S \in \mathcal{L}(H)$ then $I + S - S^*$ is invertible. (You can assume S and T are compact)

54. Please solve

$$x(s) - \int_0^s (t-s)x(t)\,dt = s\,, \qquad s \in [0,1]\,,$$

using successive approximation.