Inverse Problems and MRAI

Mapping the pulse wave velocity

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Joint work with:
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Introduction

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- Why do we want to estimate it?

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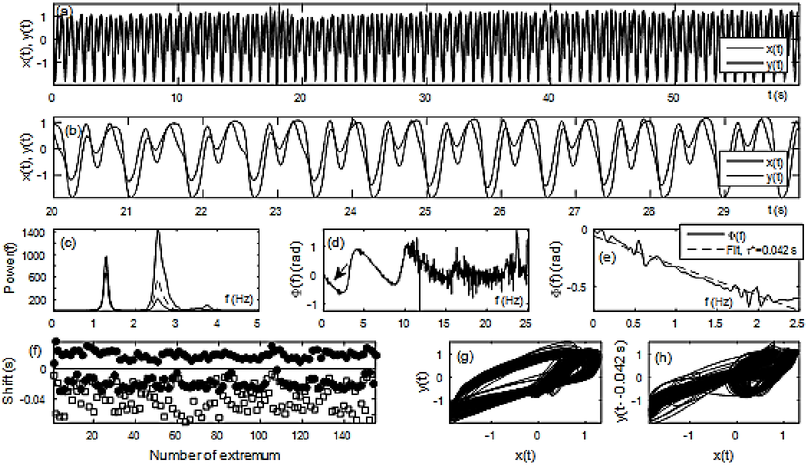
Estimate the PWV from dynamic MRI data!

Three natural questions:

- What is the PWV?
- Why do we want to estimate it?
- How can we estimate it?

The Pulse Wave



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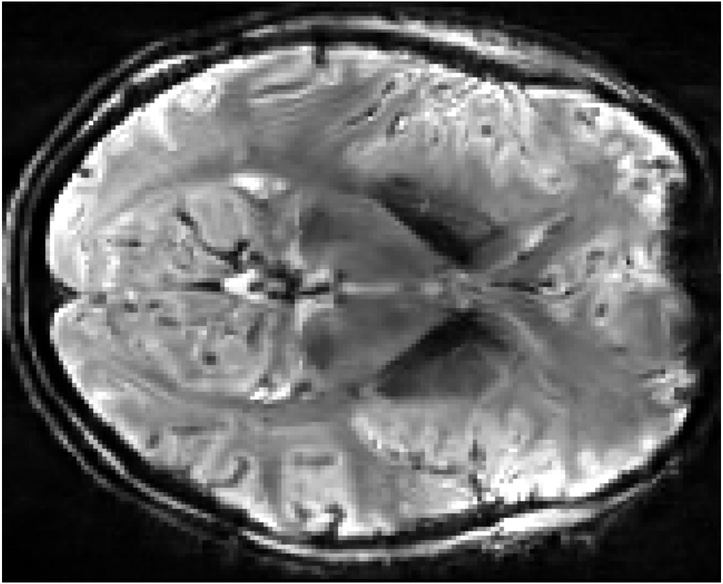
The parameters are

- h - vessel wall thickness,
- d - vascular diameter,
- E - vessel all's Young modulus,
- ρ_B - density of the blood.



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$$\boxed{\frac{\partial}{\partial t}\rho(x, y, z, t) + \operatorname{div}\left(v(x, y, z)\rho(x, y, z, t)\right) = 0\,.}$$

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MRAI = Magnetic Resonance Advection Imaging

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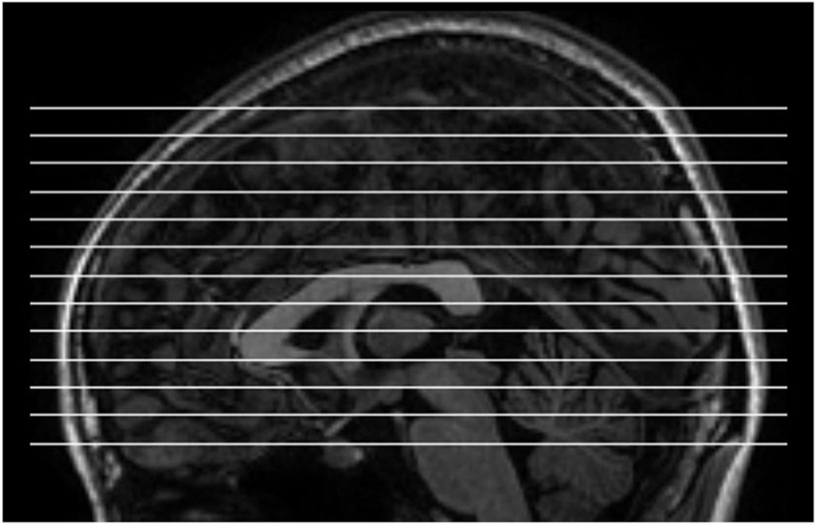
- Partial data $\longleftrightarrow$ slice-time-acquisition problem.

Slice-Time Acquisition



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Idea: Discretize the advection equation according to the data!!

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then becomes a discrete linear system of equations:

$$\begin{aligned}\frac{\rho_{i,j,k,l}-\rho_{i,j,k,l-1}}{(K+1)\Delta t} &+ D_{x_i}\rho_{i,j,k,l}\,v_{1,i,j,k}\\ &+ D_{y_j}\rho_{i,j,k,l}\,v_{2,i,j,k} + D_{z_k}\rho_{i,j,k,l}\,v_{3,i,j,k} = 0\,,\end{aligned}$$

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where

$$\rho_{i,j,k,l} = \rho(x_i, y_j, z_k, t_{k,l})\,, \quad v_{m,i,j,k} = v_m(x_i, y_j, z_k)\,, \quad m = 1,2,3\,.$$

Discretization - Finite Differences

$$D_{x_i}\rho_{i,j,k,l} := \begin{cases} \dfrac{\rho_{i+1,j,k,l} - \rho_{i-1,j,k,l}}{2\Delta x}, & 1 \leq i \leq I-1 \\ \dfrac{\rho_{1,j,k,l} - \rho_{0,j,k,l}}{\Delta x}, & i = 0 \\ \dfrac{\rho_{I,j,k,l} - \rho_{I-1,j,k,l}}{\Delta x}, & i = I \end{cases}$$

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Discretization - Finite Differences

$$
D_{z_k}\rho_{i,j,k,l} := \begin{cases}
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$$

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r := \frac{1}{K+1}
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We denote the solution $\vec{\rho}$ of this equation with $\rho(\vec{v}, \vec{\rho}_0)$.

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We define the following operator

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We can now write our problem in standard form, i.e.,

$$\boxed{\text{"}\; F\left(\vec{v}, \vec{\rho}_0\right) = \left(\vec{\rho}^{\,\delta}, \vec{\rho}_0^{\,\delta}\right) \;\text{"}}$$

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$$\Longrightarrow F(\vec{v},\vec{\rho}_0) := \left(\rho(\vec{v},\vec{\rho}_0),\vec{\rho}_0, D\vec{v}\right).$$

Choosing the matrix H

Remember the inner product:

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- **Solution 1:** Use only the diagonal entries.
- **Solution 2:** Use Wavelets instead of H.

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Landweber type gradient method:

$$x_{k+1}^{\delta} = x_k^{\delta} + \omega_k^{\delta} \, F'(x_k^{\delta})^*(y^{\delta} - F(x_k^{\delta})) \, .$$

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Discrepancy principle:

$$\left\| y^\delta - F(x_{k_*}^\delta) \right\| \leq \tau\delta \leq \left\| y^\delta - F(x_k^\delta) \right\|\,, \quad 0 \leq k \leq k_*\,.$$

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Shrinkage function:

$$S_{\tau,p}(x) = \begin{cases} \operatorname{sgn}(x) \max(|x| - \tau, 0)\,, & p = 1\,, \\ G_{\tau,p}^{-1}(x)\,, & p \in (1,2]\,, \end{cases}$$

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Resulting iteration:

$$x_{k+1}^\delta = S_{\omega_k^\delta \alpha, p}\left(x_k^\delta + \omega_k^\delta\, F'(x_k^\delta)^*(y^\delta - F(x_k^\delta)) \right)\ .$$

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Instead of using

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we can use

$$z_k^\delta = x_k^\delta + \tfrac{k-1}{k+2}(x_k^\delta - x_{k-1}^\delta),$$
$$x_{k+1}^\delta = S_{\omega_k^\delta \alpha, p}\left(z_k^\delta + \omega_k^\delta F'(z_k^\delta)^*(y^\delta - F(z_k^\delta))\right).$$

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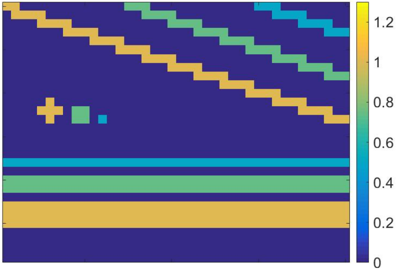
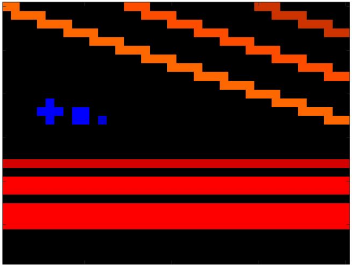
$\Longrightarrow$ *Runs on a standard home computer in acceptable time!!!*

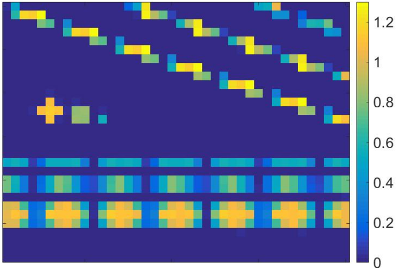
(Real-world data set has 3 million unknowns)

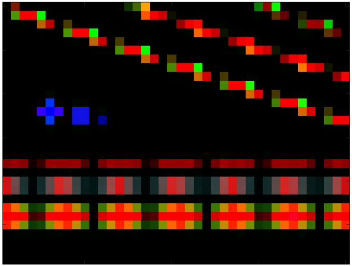
Simulation Phantom - MIP and direction MIP



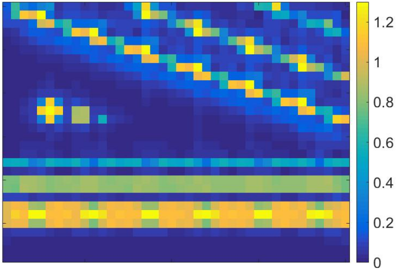


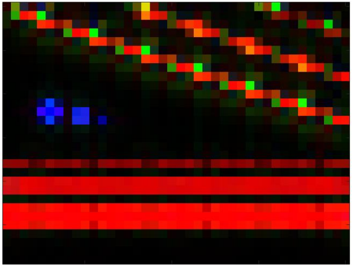
Results - Pure Method



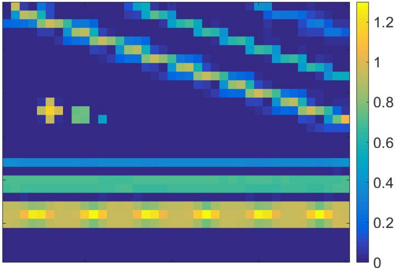


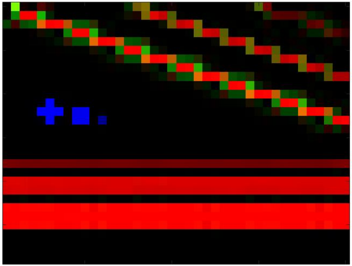
Results - Divergence-Free





Results - Divergence-Free + Wavelets + Sparsity





Natural Stimulation Data Set

Natural Stimulation Data Set

Specifications:

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- Publicly available natural stimulation dynamic EPI data.

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- Data has dimension $132 \times 175 \times 48$.

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Natural Stimulation Data Set

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- Pulse repetition time (TR) of 2 seconds.
- Eight 15 minutes long segments for each subject.

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Algorithm specifics:

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- First 20 seconds of second segment were used.

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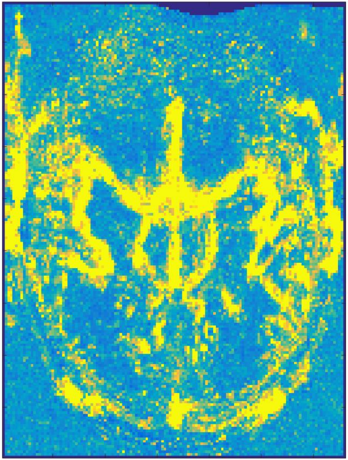
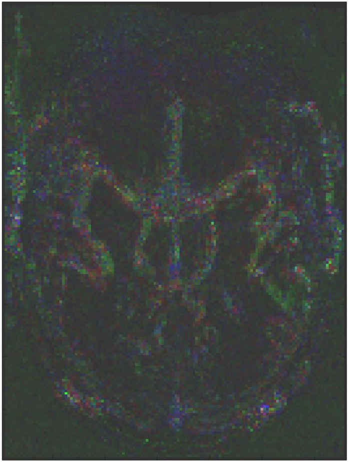
- Eight 15 minutes long segments for each subject.

Algorithm specifics:

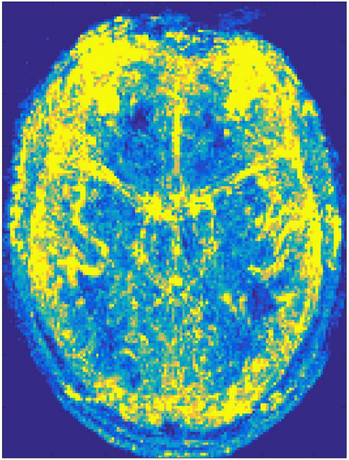
- First 20 seconds of second segment were used.
- **Stopping rule:** Residual decrease check.

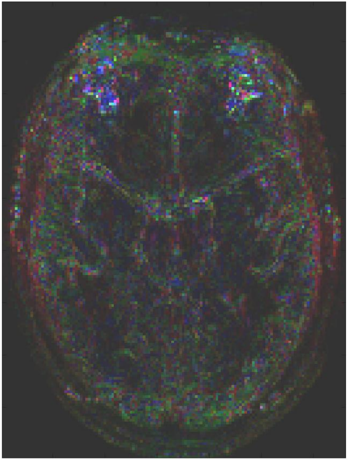
Regression Approach - Results



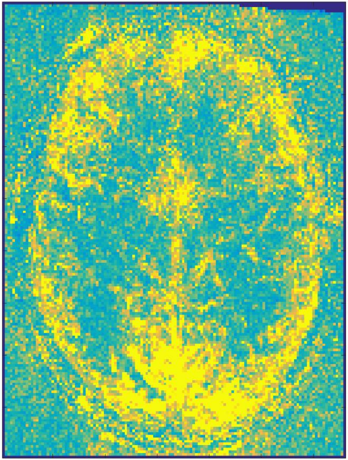


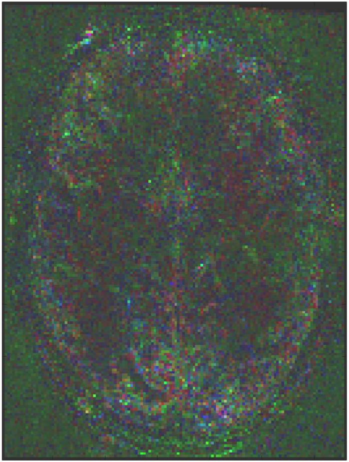
New Approach - Results



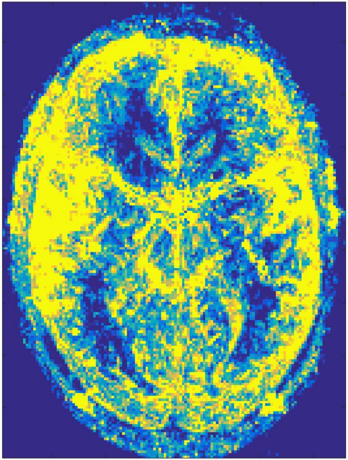


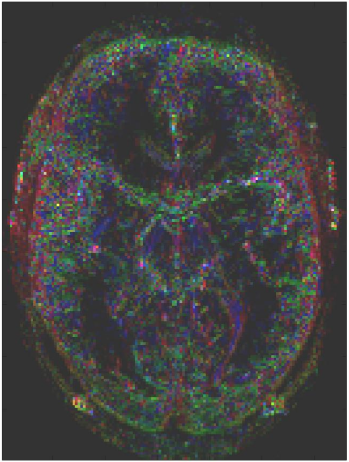
Regression Approach - Results





New Approach - Results





End

Thank you for your attention!