



Domain Decomposition Solvers for Frequency-Domain Finite Element Equations

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NuMa-Report No. 2009-13

December 2009

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Domain Decomposition Solvers for Frequency-Domain Finite Element Equations

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Summary. The paper is devoted to fast iterative solvers for frequency-domain finite element equations approximating linear and nonlinear parabolic initial boundary value problems with time-harmonic excitations. Switching from the time domain to the frequency domain allows us to replace the expensive time-integration procedure by the solution of a simple linear elliptic system for the amplitudes belonging to the sine- and to the cosine-excitation or a large nonlinear elliptic system for the Fourier coefficients in the linear and nonlinear case, respectively. The fast solution of the corresponding linear and nonlinear system of finite element equations is crucial for the competitiveness of this method.

1 Introduction

In many practical applications, for instance, in electromagnetics and mechanics, the excitation is time-harmonic. Switching from the time domain to the frequency domain allows us to replace the expensive time-integration procedure by the solution of a simple elliptic system for the amplitudes. This is true for linear problems, but not for nonlinear problems. However, due to the periodicity of the solution, we can expand the solution in a Fourier series. Truncating this Fourier series and approximating the Fourier coefficients by finite elements, we arrive at a large-scale coupled nonlinear system for determining the finite element approximation to the Fourier coefficients. In the literature, this approach is called multiharmonic FEM or harmonic-balanced FEM, and has been used by many engineers in different applications. see, e.g., Bachinger et al. [2002] and the references therein.

Bachinger et al. [2005] provided the first rigorous numerical analysis for the eddy current problem. The practical aspects of the multiharmonic approach, including the construction of a fast multigrid preconditioned QMR solver for the Jacobi system arising in every Newton step and the implementation in

an adaptive multilevel setting, are discussed in Bachinger et al. [2006] by the same authors. There was no rigorous analysis of the multigrid preconditioned QMR solver, but the numerical results presented in this paper for academic and more practical problems indicated the efficiency of this solver.

The construction of fast solvers for such systems is very crucial for the overall efficiency of this multiharmonic approach. In this paper, we look at linear and nonlinear, time-harmonic potential problems. We construct and analyze an almost optimal preconditioned GMRes solver for the Jacobi systems arising from the Newton linearization of the large-scale coupled nonlinear system. This preconditioner is not robust with respect to the excitation frequency. In the linear case we are able to construct a robust preconditioner used in a Min-Res solver. The multiharmonic approach is presented in Section 2, whereas the two different preconditioners and solvers are discussed in Section 3 and Section 4.

2 Frequency-Domain Finite Element Equations

Let us consider the following nonlinear, parabolic, scalar potential equation with a homogeneous Dirichlet boundary condition and an inhomogeneous initial condition as the model problem:

$$\begin{cases} \alpha \frac{\partial u}{\partial t} - \nabla \cdot (\nu(|\nabla u|)\nabla u) = f & \text{in } \Omega \times (0, T], \\ u(\mathbf{x}, 0) = u_0(\mathbf{x}) & \text{for } \mathbf{x} \in \bar{\Omega}, \\ u(\mathbf{x}, t) = 0 & \text{for } (\mathbf{x}, t) \in \partial\Omega \times [0, T], \end{cases} \quad (1)$$

where the right-hand side $f(x, t)$ is given by a time-harmonic excitation with the frequency ω , i.e.

$$f(\mathbf{x}, t) = f^c(\mathbf{x}) \cos(\omega t) + f^s(\mathbf{x}) \sin(\omega t). \quad (2)$$

We assume that $\Omega \subset \mathbb{R}^3$ is a bounded Lipschitz domain, α is a given uniformly positive function in $L_\infty(\Omega)$, and $\nu : \mathbb{R}_0^+ \rightarrow \mathbb{R}^+$ is a continuously differentiable function satisfying the properties

$$0 < \nu_{\min} \leq \nu(s) \leq \nu_{\max} \quad \text{for } s \geq 0, \quad (3)$$

$$s \mapsto s\nu(s) \quad \text{is Lipschitz and strongly monotone for } s \geq 0. \quad (4)$$

These conditions ensure that there exists at least a unique weak solution to the initial boundary value problem (1), see Zeidler [1990]. In the linear case where the coefficient ν is independent of $|\nabla u|$, the solution $u(\mathbf{x}, t) = u^c(\mathbf{x}) \cos(\omega t) + u^s(\mathbf{x}) \sin(\omega t)$ is time-harmonic as well, and we get an elliptic boundary value problem for defining the unknown amplitudes u^c and u^s which only depend on the spatial variable \mathbf{x} . This is not true in the nonlinear case. However, the solution u to (1) is still periodic in time, with frequency ω . Thus, we have the Fourier series representation

$$\mathbf{F}_h(\mathbf{u}_h) = \mathbf{f}_h \quad (7)$$

for determining the finite element solution

$$\mathbf{S}_h^1 := \left(\text{span}\{\phi_j\}_{j=1}^{N_h} \right)^{2N} \ni \tilde{\mathbf{u}}_h \leftrightarrow \mathbf{u}_h = (\underline{u}_{1,h}^c, \underline{u}_{1,h}^s, \dots, \underline{u}_{N,h}^c, \underline{u}_{N,h}^s)^T \in \mathbb{R}^{2N \cdot N_h}$$

to the Fourier coefficients $\mathbf{u} \in H_0^1(\Omega)^{2N}$. Here, ϕ_j are piecewise linear basis functions in $H_0^1(\Omega)$. Thus the multiharmonic approach yields a time-independent nonlinear system for the solution of which highly parallel solvers can be constructed.

Following Bachinger et al. [2005] we can show that under standard regularity assumptions, the discretization error behaves like $O(h + N^{-1})$ with respect to the $L((0, T), H^1(\Omega))$ norm.

Solving (7) by Newton's method ($\tau_n = 1$)

$$\mathbf{u}_h^{n+1} = \mathbf{u}_h^n + \tau_n \mathbf{w}_h^n = \mathbf{u}_h^n + \tau_n \mathbf{F}'_h(\mathbf{u}_h^n)^{-1}(\mathbf{f}_h - \mathbf{F}_h(\mathbf{u}_h^n)), \quad (8)$$

we have to solve the large-scale linear system

$$\mathbf{F}'_h(\mathbf{u}_h^n) \mathbf{w}_h^n = \mathbf{r}_h^n := \mathbf{f}_h - \mathbf{F}_h(\mathbf{u}_h^n), \quad (9)$$

with the Jacobi matrix $\mathbf{F}'_h(\mathbf{u}_h^n)$ as system matrix and the residual \mathbf{r}_h^n as right-hand side.

Copeland and Langer [2009] show that the Jacobi-systems (9) can successfully be solved by the preconditioned GMRes method using a special domain decomposition preconditioner. We will explain the construction of this preconditioner for the corresponding linear problem in the next section, but the results remain valid for the Jacobi-systems (9) too.

In the remainder of this paper, we discuss preconditioned iterative methods for solving linear systems of the form

$$\begin{pmatrix} K_h & \sigma M_h \\ -\sigma M_h & K_h \end{pmatrix} \begin{pmatrix} \underline{u}_h^c \\ \underline{u}_h^s \end{pmatrix} = \begin{pmatrix} \underline{f}_h^c \\ \underline{f}_h^s \end{pmatrix}, \quad (10)$$

arizing from the time-domain finite element discretization of the initial-boundary value problem (1) with the time-harmonic excitation (2) in the linear case where the coefficient ν is independent of $|\nabla u|$. The coefficient σ is equal to $\alpha\omega$. Here and in the following, we assume that α is a positive constant. The stiffness matrix K_h and the mass matrix M_h are computed from the bilinear forms

$$\int_{\Omega} \nu(\mathbf{x}) \nabla \phi(\mathbf{x}) \cdot \nabla \psi(\mathbf{x}) \, d\mathbf{x} \quad \text{and} \quad \int_{\Omega} \phi(\mathbf{x}) \psi(\mathbf{x}) \, d\mathbf{x},$$

respectively. The system matrix \mathbf{D}_h in (10) is obviously positive definite and non-symmetric (block skew-symmetric).

3 Domain Decomposition Solver

Following Xu and Cai [1992], we propose a non-symmetric two-level Schwarz preconditioner for (10) of the form

$$\mathbf{C}_h^{-1} = \mathbf{I}_H^h \mathbf{D}_H^{-1} \mathbf{I}_h^H + \beta \mathbf{B}_h^{-1}, \quad (11)$$

where \mathbf{D}_H is a coarse grid version of \mathbf{D}_h , \mathbf{I}_h^H and \mathbf{I}_H^h are appropriate restriction and prolongation operators, \mathbf{B}_h is a symmetric positive definite (SPD) preconditioner for the SPD part $\mathbf{A}_h = \text{blockdiag}(K_h, K_h)$ of \mathbf{D}_h , and β is a positive scaling constant. Dryja et al. [1994] proposed a wire-basket-based domain decomposition method that gives an effective preconditioner \mathbf{B}_h for the symmetric positive definite matrix \mathbf{A}_h , with a condition number estimate which is independent of jumps in the coefficient and depends only polylogarithmically on H/h , see also Toselli and Widlund [2005]. Using this wire-basket domain decomposition preconditioner \mathbf{B}_h in (11), we arrive at the following convergence estimate for the GMRes preconditioned by the XuCai-preconditioner (11):

Theorem 1 *Assume that the adjoint linear problem is $H^{1+s}(\Omega)^2$ -coercive with some $s \in (0, 1]$, and H is sufficiently small, specifically $H^s < c(1 + \log(H/h))^{-2}$. Then the GMRes method preconditioned by the preconditioner (11) with the wire-basket component \mathbf{B}_h converges and the convergence estimate*

$$\|\mathbf{r}_h^m\|_{\mathbf{A}_h} \leq \left(1 - c c_{\log}^{-4} (1 + c_{\log}^2)^{-2}\right)^{m/2} \|\mathbf{r}_h^0\|_{\mathbf{A}_h} := \gamma (H/h)^{m/2} \|\mathbf{r}_h^0\|_{\mathbf{A}_h}$$

holds for the preconditioned residual $\mathbf{r}_h^m = \mathbf{C}_h^{-1}(\mathbf{f}_h - \mathbf{D}_h \mathbf{u}_h^m)$ at the m -th iteration, where $c_{\log} := 1 + \log(H/h)$, $0 < \gamma(H/h) < 1$, and the constant c depends on ν and σ , but not on H and h .

The proof of this theorem can be found in Copeland and Langer [2009]. In the same paper we present our numerical results which show that our preconditioned GMRes method is a quite efficient solver for the linear system (10) and can efficiently be used for solving the Jacobi-systems (9) as well. The number of iterations only polylogarithmically depends on H/h . In order to clarify the dependence on σ , Kolmbauer [2009] performed a Fourier analysis of the preconditioned matrix $\mathbf{C}_h^{-1} \mathbf{D}_h$ for the one-dimensional problem with constant ν , where the exact SPD part \mathbf{A}_h was used as \mathbf{B}_h , and $H = 2h$. This analysis shows that this preconditioner is not robust with respect to σ , see also the second line of Table 2. In the next section we present a robust preconditioner for the linear system (10) in an equivalent symmetric, but indefinite setting.

4 A Symmetric and Indefinite Reformulation

The non-symmetric and positive definite system (10) can be reformulated in the following equivalent form

$$\begin{pmatrix} M_h & K_h \\ K_h & -\sigma^2 M_h \end{pmatrix} \begin{pmatrix} \underline{u}_h^s \\ \frac{1}{\sigma} \underline{u}_h^c \end{pmatrix} = \begin{pmatrix} \frac{1}{\sigma} \underline{f}_h^c \\ \underline{f}_h^s \end{pmatrix} \quad (12)$$

with a symmetric but indefinite system matrix \mathbf{D}_h . For simplicity, we use the same notation \mathbf{D}_h for the system matrix in (10) and (12). It follows from Schöberl and Zulehner [2007] that the block-diagonal preconditioner

$$\mathbf{C}_h = \frac{1}{\sigma} \begin{pmatrix} \sigma M_h + K_h & 0 \\ 0 & -\sigma^2(\sigma M_h + K_h) \end{pmatrix} \quad (13)$$

is robust with respect to both the discretization parameter h and the bad parameter σ . More precisely, the condition number

$$\kappa(\mathbf{C}_h^{-1} \mathbf{D}_h) = \|\mathbf{C}_h^{-1} \mathbf{D}_h\|_{\mathbf{C}_h} \|\mathbf{D}_h^{-1} \mathbf{C}_h\|_{\mathbf{C}_h} = |\lambda_{2N_h}|/|\lambda_1| \leq c = \text{const} \quad (14)$$

can be estimated by a positive constant c that is independent of both h and σ , where the eigenvalues of the preconditioned matrix $\mathbf{C}_h^{-1} \mathbf{D}_h$ are ordered in such a way that $|\lambda_{2N_h}| \geq |\lambda_{2N_h-1}| \geq \dots \geq |\lambda_1| > 0$. Therefore, solving

$$\mathbf{C}_h^{-1} \mathbf{D}_h \underline{\mathbf{u}}_h = \mathbf{C}_h^{-1} \underline{\mathbf{f}}_h$$

by means of the MinRes method proposed by Paige and Saunders [1975], we can ensure that the preconditioned residual $\underline{\mathbf{r}}_h^{2m} = \mathbf{C}_h^{-1} \underline{\mathbf{f}}_h - \mathbf{C}_h^{-1} \mathbf{D}_h \underline{\mathbf{u}}_h^{2m}$ of the $2m$ -th MinRes iterate satisfies the iteration error estimate

$$\|\underline{\mathbf{r}}_h^{2m}\| \leq \frac{2q^m}{1 - q^{2m}} \|\underline{\mathbf{r}}_h^0\| \quad (15)$$

with $q = (\kappa(\mathbf{C}_h^{-1} \mathbf{D}_h) - 1)/(\kappa(\mathbf{C}_h^{-1} \mathbf{D}_h) + 1)$, see e.g. Voss [1993] or Greenbaum [1997]. Thus, the number of MinRes iterations required for reducing the initial error by some fixed factor $\varepsilon \in (0, 1)$ is independent of both h and σ . Of course, in practice, the diagonal blocks $\sigma M_h + K_h$ in the preconditioner (13) should be replaced by appropriate preconditioners, e.g. by appropriate domain decomposition or multigrid preconditioners, see e.g. Toselli and Widlund [2005].

Applying again the Fourier analysis (FA) to our one-dimensional problem gives quantitative rates which are displayed in Table 1, for σ ranging from 10^{-10} to 10^{10} .

Table 2 provides the MinRes iteration numbers which are needed for reducing the initial error by the factor $\varepsilon = 10^{-5}$ for different h and σ . The second line contains the preconditioned GMRes iterations for the constellation $h = 1/60$ and $H = 1/10$, where we use the preconditioner (11) with $\mathbf{B}_h = \mathbf{A}_h$. Both the FA (Table 1) and the numerical experiments (Table 2) were performed for the one-dimensional linear problem resulting in the stiffness matrix $K_h = h^{-1} \text{tridiag}(-1, 2, -1)$ and in the mass matrix $M_h = (h/6) \text{tridiag}(1, 4, 1)$ for the case $\nu = 1$. However, due to the estimates (14) and (15) the numerical behavior observed in our one-dimensional example is characteristic for the three-dimensional linear problem as well.

Table 1. Convergence rate q resulting from the FA ($\varepsilon = 10^{-5}$).

$\log_{10}\sigma$	-10	-8	-6	-4	-2	0	2	4	6	8	10
$h = 1/60$	$< \varepsilon$	$< \varepsilon$	$< \varepsilon$	$< \varepsilon$	0.0005	0.046	0.17	0.17	0.021	0.0002	$< \varepsilon$
$h = 1/120$	$< \varepsilon$	$< \varepsilon$	$< \varepsilon$	$< \varepsilon$	0.0005	0.046	0.17	0.17	0.072	0.0009	$< \varepsilon$
$h = 1/1200$	$< \varepsilon$	$< \varepsilon$	$< \varepsilon$	$< \varepsilon$	0.0005	0.046	0.17	0.17	0.17	0.072	0.0009
$h = 1/12000$	$< \varepsilon$	$< \varepsilon$	$< \varepsilon$	$< \varepsilon$	0.0005	0.046	0.17	0.17	0.17	0.17	0.072
$h = 1/120000$	$< \varepsilon$	$< \varepsilon$	$< \varepsilon$	$< \varepsilon$	0.0005	0.046	0.17	0.17	0.17	0.17	0.17

Table 2. Number of GMRes and MinRes iterations for $\varepsilon = 10^{-5}$.

$\log_{10}\sigma$	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
GMRes	1	1	1	1	1	1	2	2	2	3	4	5	9	18	36	52	52	52	52	52	52
$h = 1/60$	1	1	1	1	1	3	3	3	5	7	11	13	13	14	10	6	4	4	4	2	2
$h = 1/120$	1	1	1	1	1	3	3	3	5	7	11	13	13	14	12	8	4	4	4	2	2
$h = 1/1200$	1	1	1	1	1	3	3	3	5	7	11	15	13	14	13	12	10	6	4	4	4
$h = 1/12000$	1	1	1	1	1	3	3	3	5	7	11	15	13	14	13	12	12	11	10	6	6
$h = 1/120000$	1	1	1	1	1	3	3	3	5	7	11	15	13	14	13	12	12	11	10	10	10

5 Conclusions, Outlook, and Acknowledgments

In this paper we have considered the harmonic and multiharmonic approach to the solution of linear and nonlinear parabolic initial-boundary value problems with harmonic excitation. We have proposed two solution strategies based on a preconditioned GMRes method for the positive definite and non-symmetric problem formulation and a preconditioned MinRes iteration method for the symmetric and indefinite reformulation of the problem. The preconditioner for the GMRes method is a two-level Schwarz preconditioner consisting of a coarse grid solver for the original non-symmetric problem and a wire-basket-based domain decomposition preconditioner for the SPD part. This iterative solver works well for both the linear system (10) arising from the linear time-harmonic problem and the Jacobi-systems (9) arising in every step of the Newton iteration (8) for solving the nonlinear equations (7). This preconditioner is highly parallel, but not robust with respect to the bad parameter σ . A robust preconditioner can be constructed for the linear case where ν is independent of $|\nabla u|$. The preconditioner used in the MinRes method has a block-diagonal structure and is robust with respect to both the discretization parameter h and the bad parameter σ . Of course, other iterative methods are possible like the symmetric Uzawa CG method considered in Schöberl and Zulehner [2007] or the QMR method used in Bachinger et al. [2006]. Furthermore, the robust all-at-once multigrid solvers developed by Schöberl et al. [2009] for solving saddle point problems can be an alternative to the preconditioned Krylov-subspace methods considered in this paper. The preconditioned GMRes and MinRes solvers presented in this paper can be generalized to nonlinear eddy current problems studied in Bachinger et al. [2005] and Bachinger et al. [2006].

The authors gratefully acknowledge the financial support by the Austrian Science Fund (FWF) under the grant P19255 and by the Award No. KUS-C1-016-04, made by King Abdullah University of Science and Technology.

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