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Topology Optimization of Electric Motor Using Topological Derivative for Nonlinear Magnetostatics

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We aim at finding an optimal design for an interior permanent magnet electric motor by means of a sensitivity-based topology optimization method. The gradient-based ON/OFF method, introduced by Y. Okamoto and N. Takahashi in 2005, has been successfully applied to optimization problems of this form. We show that this method can be improved by considering the mathematical concept of topological derivatives. Topological derivatives for optimization problems constrained by linear partial differential equations (PDEs) are well-understood, whereas little is known about topological derivatives in combination with nonlinear PDE constraints. We derive the topological derivative for an optimization problem constrained by the equation of nonlinear two-dimensional magnetostatics, illustrate its advantages over the sensitivities used in the ON/OFF method, and show numerical results for the optimization of an interior permanent magnet electric motor obtained by a level-set algorithm.

***Index Terms*—design optimization, permanent magnet motors, rotating machines, sensitivity analysis**

I. Introduction

Topology optimization methods originate from mechanical engineering [1], but have found more and more applications in electromagnetics in recent years. They aim at finding designs which are optimal with respect to some criteria. More precisely, they seek for the distribution of material in a design subdomain that minimizes a given design-dependent objective functional J. In contrast to shape optimization methods, where only the boundary or interface of a structure can be modified, topology optimization methods also allow for a nucleation of holes and thus for a change of the topology of the initial design. Therefore, they are much more flexible and often yield better results than shape optimization methods.

A useful tool for topology optimization are topological sensitivities. At each point of the design subdomain, they indicate whether a local change of the material would increase or decrease the objective functional. The ON/OFF method, was introduced in [2] and adopted to a sensitivity-based method in [3], which uses information about the sensitivity of the cost functional with respect to a local perturbation of the magnetic reluctivity to improve the design of a given device. It was successfully applied to the optimization of various electromagnetic devices [4-7]. The mathematical concept of topological derivatives is based on the idea of [8] and was introduced in a mathematically rigorous way in [9]. It was shown in [10] that, in the case of a linear state equation, the sensitivity used in the ON/OFF method is equivalent to the topological derivative. For the optimization of electrical machines, we are faced with a nonlinear state equation. In this case, the application of the ON/OFF method generalizes directly, whereas the formula for the topological derivative has been an open problem.

In this work, we compare the ideas of the ON/OFF method and the topological derivative and show that the sensitivity used in the ON/OFF method is not the right quantity to be considered for topology optimization. We present the formula for the topological derivative in the case of nonlinear material behavior and show numerical results for a model problem that were obtained by application of a level-set algorithm [11].

II. Problem Description

We consider an interior permanent magnet (IPM) brushless electric motor as depicted in Fig. 1 that consists of ferromagnetic material, permanent magnets, coil areas and air regions. For our special application we do not consider any electric current induced in the coils. Our goal is to find a design that yields a small total harmonic distortion (THD) of the radial component of the magnetic flux density in the air gap, while retaining a large amplitude of its first harmonic. We search for the distribution of ferromagnetic material in the design regions of the rotor (striped areas in Fig. 1) that minimizes the ratio between the THD and this amplitude. The optimization problem looks as follows:

$$J(u(\Omega)) = \frac{\mathrm{THD}\left(\mathbf{B}_r(u(\Omega))\right)}{A_1\left(\mathbf{B}_r(u(\Omega))\right)} \to \min \qquad (1)$$

$$\text{s.t.} \begin{cases} -\mathrm{div}\,(\nu(|\nabla u|)\,\nabla u - \mathbf{M}^{\perp}) & = \quad 0 \quad \text{in } \Omega \\ \qquad u & = \quad 0 \quad \text{on } \partial\Omega \end{cases} \qquad (2)$$

Here, we consider the setting of two-dimensional magnetostatics where the magnetic flux density is only acting in the x_1-x_2 plane, i.e., $\mathbf{B} = (B_1, B_2, 0)^T$, and $\Omega \subset \mathbb{R}^2$ is a bounded Lipschitz domain as depicted in Fig. 1. The state variable $u = u(x_1, x_2)$ denotes the third component of the magnetic vector potential, i.e., $\mathbf{B}(u) = \mathrm{curl}((0,0,u)^T)$, and $\mathbf{B}_r$ the radial component of $\mathbf{B}$ in the air gap. Note that the magnetic vector potential $(0,0,u)^T$ satisfies the Coulomb gauge condition. Further, $\mathbf{M}^{\perp} = (-M_2, M_1)^T$ denotes the perpendicular of the permanent magnetization which vanishes outside the permanent magnets, and ν represents the magnetic reluctivity, which is a nonlinear function $\hat{\nu}$ in the ferromagnetic subdomain Ω_f (brown area in Fig. 1) and the constant ν_0 (magnetic reluctivity of air) else, i.e.,

$$\nu(|\nabla u|) = \begin{cases} \hat{\nu}(|\nabla u|) & x \in \Omega_f, \\ \nu_0 & x \in \Omega_{air}. \end{cases} \qquad (3)$$

We will later refer to a simplified linear setting where the nonlinear function $\hat{\nu}$ is replaced by some constant ν_1. Note that J depends on Ω_f via the solution u of the state equation (2). The THD of $\mathbf{B}_r$ is defined via the Fourier coefficients,

$$\mathbf{B}_r = \sum_{k=1}^{\infty} A_k \cos(\omega k x), \ \ \mathrm{THD}(\mathbf{B}_r) = \sqrt{\left(\sum_{k=2}^{N} A_k^2\right) \Big/ \left(\sum_{k=1}^{N} A_k^2\right)},$$

and here serves as a measure of the smoothness of the rotation of the rotor.

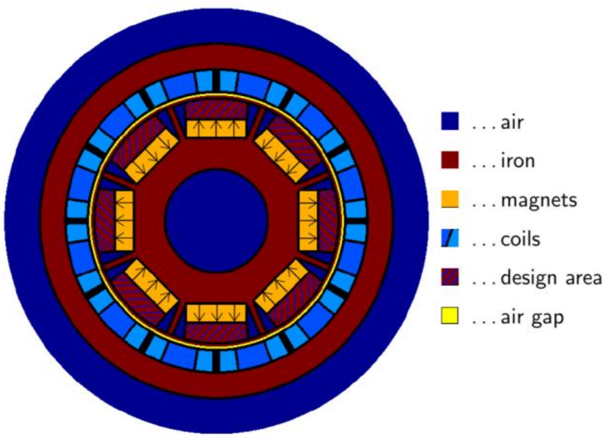

Fig. 1. Computational domain Ω representing electric motor with different subdomains

III. ON/OFF Method

The sensitivity-based ON/OFF method as introduced in [3] is based on the fact that the difference between having iron or air in a spatial point is only reflected in the value of the magnetic reluctivity ν which is a constant ν_0 in the air subdomain and a nonlinear function $\hat{\nu}$ in the ferromagnetic material. This nonlinear function $\hat{\nu}$ usually attains values that are much smaller than ν_0. The idea is to compute the sensitivity of the objective function with respect to a local perturbation of the magnetic reluctivity in one element of the finite element (FE) mesh, i.e.,

$$\frac{dJ}{d\nu_k},$$

where ν_k is the magnetic reluctivity in element T_k. Whenever this sensitivity is negative, assuming monotonicity of J with respect to ν_k, a larger value for ν_k would yield a smaller value for J, which is realized by switching the element OFF, i.e., by setting it to air. On the other hand, if the sensitivity is positive, switching the element ON, i.e., setting it to iron, would be favorable for reducing the cost functional J. We remark that this sensitivity has an intrinsic dependence on the size of the elements in the FE mesh which can be avoided by scaling the sensitivities by the element size. For piecewise linear FE calculations, the resulting sensitivities read

$$S_k := \frac{1}{|T_k|} \frac{dJ}{d\nu_k} = U_0^T \, V_0 \qquad (4)$$

where $U_0 = \nabla u$ and $V_0 = \nabla \mathrm{v}$ restricted to the element T_k. Here, v is the solution to the adjoint equation of problem (1)-(2). Note that formula (4) is the same for the linear and the nonlinear case, the only difference lies in the definition of the adjoint state v. For the definition of the adjoint equation, we refer the reader to [10].

This method has, amongst other applications, been successfully applied to the optimization of electromagnetic shielding [4], [5] and electric motors [6]. However, it is based on heuristics for the following reason: The sensitivity S_k just gives information about the behavior of J for a small variation of the magnetic reluctivity. When changing the material, the reluctivity is switched from $\hat{\nu}$ directly to ν_0 (or vice versa). The correct quantity to be considered for this scenario is the topological derivative.

IV. Topological Derivative

A. Definition

The concept of the topological derivative (TD) came up as the bubble method in [8] and was introduced in a mathematically rigorous way in [9]. The topological derivative of a domain-dependent functional $J = J(\Omega)$ at a point x_0 describes its sensitivity with respect to a perturbation of the domain in a neighborhood of that point. It is defined as the quantity $G(x_0)$ satisfying a topological asymptotic expansion of the form

$$J(\Omega_\varepsilon) - J(\Omega) = \varepsilon^d \, G(x_0) + o(\varepsilon^d) \quad \text{as } \varepsilon \to 0. \qquad (5)$$

Here, Ω_ε denotes the perturbed configuration where the material property in a small neighborhood $\omega_\varepsilon = x_0 + \varepsilon D$ (with D, e.g., the unit disk) of the spatial point x_0 is switched from iron to air or vice versa, and d is the space dimension (here: $d = 2$). Note that, when $G(x_0)$ is negative for a point x_0, this definition yields that, for ε small enough, switching the material in ω_ε decreases the objective functional.

B. Formulas

We want to emphasize that we get two different formulas for the topological derivative depending on whether we are interested in the sensitivity for a local transition from ferromagnetic material to air or vice versa. We will denote them by $G^{f\to air}$ and $G^{air\to f}$, respectively. Let v the adjoint state of optimization problem (1)-(2). Furthermore, let $U_0 = \nabla u(x_0)$ and $V_0 = \nabla \mathrm{v}(x_0)$.

1) Linear case

It was shown in [10] that in the simplified linear setting, where the nonlinear function $\hat{\nu}$ in (2) is replaced by a constant ν_1, the formulas for the topological derivative for introducing air in iron of optimization problem (1)-(2) reads

$$G^{f\rightarrow \mathrm{air}}(x_0) = C^{f\rightarrow \mathrm{air}}\, U_0^T\, V_0 \tag{6}$$

with $C^{f\rightarrow air} = 2\pi(\nu_0 - \nu_1)/(\nu_0 + \nu_1)$. The formula for $G^{air\rightarrow f}$ can be obtained by switching the roles of ν_0 and ν_1. We get

$$G^{\mathrm{air}\rightarrow f}(x_0) = C^{\mathrm{air}\rightarrow f} U_0^T\, V_0, \tag{7}$$

with $C^{air\rightarrow f} = -(\nu_0/\nu_1)\, C^{f\rightarrow air}$.

2) Nonlinear case

In the nonlinear case, the topological derivative of optimization problem (1)-(2) is a sum of two terms. The derivation of this lengthy formula is similar to [12]. In both cases, the TD has the form

$$G(x_0)=\mathrm{U}_0^T\, P(U_0)\, V_0 + \int S_{U_0}(\nabla H)(V_0 + \nabla K)$$

where $P(U_0)$ denotes a polarization matrix, H is the variation of the direct state and K the variation of the adjoint state, both after a change of scale. The functions H and K are solutions to a nonlinear and a linear partial differential equation (PDE) on the entire plane $\mathbb{R}^2$. The difference between $G^{f\rightarrow air}$ and $G^{air\rightarrow f}$ lies in the definitions of S_{U_0}, H, K as well as of the polarization matrix $P(U_0)$. The polarization matrices for the two cases read

$$P^{f\rightarrow \mathrm{air}}(U_0) = \mathrm{C}_1 R_\varphi \begin{pmatrix} \dfrac{1}{1+\nu_0\, e/(\alpha+\beta)} & 0 \\ 0 & \dfrac{1}{\mathrm{e+r}} \end{pmatrix} R_\varphi^T,$$

$$P^{\mathrm{air}\rightarrow f}(U_0) = \mathrm{C}_2 R_\varphi \begin{pmatrix} \dfrac{r-1}{(\mathrm{r}+1)(r-1-\beta/\alpha)} & 0 \\ 0 & \dfrac{1}{\mathrm{r}+1} \end{pmatrix} R_\varphi^T,$$

where $C_1 = (\nu_0 - \alpha)\pi(1+e)$, $C_2 = 2r(\alpha - \nu_0)\pi$, $r = \nu_0/\alpha$, $e = \alpha/(\alpha+\beta)$, $\alpha = \hat{\nu}(|U_0|), \beta = \hat{\nu}'(|U_0|)$, φ the angle between U_0 and the x-axis and R_φ the rotation matrix around angle φ.

V. Comparison

Comparing formulas (4) and (6) for the linear case, we see that they only differ by a constant factor $C^{f\rightarrow air}$. When one is only interested in removing material, this factor is of no importance. However, when one wants to design a bidirectional optimization method that can decide whether it is better to remove material at one place or to add material at another, the information contained in these constants $C^{f\rightarrow air}$ and $C^{air\rightarrow f}$ is crucial.

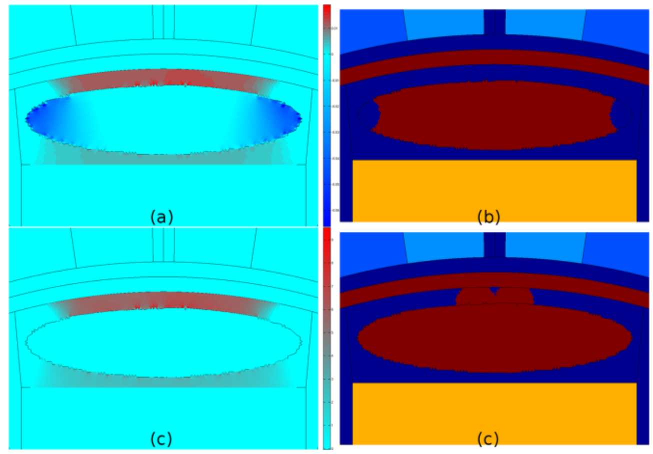

Fig. 2. (a) sensitivities S_k for ellipse-shaped design (b) Design after removing material according to sensitivities S_k, J(u) = 0.436 (c) generalized topological derivative for ellipse-shaped design (d) design after adding material according to toplogical derivative, J(u) = 0.406

In the nonlinear case, we observe that the topological derivative differs from the scaled version of the ON/OFF sensitivities, S_k, in two ways: on the one hand, in the polarization matrices $P^{f\rightarrow air}(U_0)$ and $P^{air\rightarrow f}(U_0)$, which play the same role as the constants $C^{f\rightarrow air}$ and $C^{air\rightarrow f}$ in the linear case, but now depend on the gradient of the solution u at point x_0; on the other hand, in the presence of a new second term which accounts for the nonlinearity of the problem. Numerical experiments showed that, in regions of low flux density, the second term in (8) is negligible in comparison with the first term. Therefore, we will neglect this term for the rest of this work. In Fig. 2, we illustrate the importance of the polarization matrices $P^{f\rightarrow air}(U_0)$ and $P^{air\rightarrow f}(U_0)$. For a hypothetical ellipse-shaped design, the sensitivities S_k suggest to remove material on the left and right ends of the ellipse, whereas the TD suggests to add material at the top. Fig. 2 gives numerical evidence that the latter yields a much larger decrease of the objective functional. This phenomenon might lead to final designs obtained by the ON/OFF method that are not optimal because the wrong sensitivities have been used.

The conceptual difference between the two kinds of sensitivities is the following: The sensitivity S_k represents the sensitivity with respect to a small perturbation of the material property. This information is important, for instance, when one is interested in the sensitivity of a design with respect to manufacturing errors (see, e.g., [13]). The TD is the sensitivity with respect to a change of material from $\hat{\nu}$ directly to ν_0 in a small neighborhood of a point x_0 and is therefore the right sensitivity for topology optimization.

VI. Numerical Results

Knowing both sensitivities $G^{f\rightarrow air}$ and $G^{air\rightarrow f}$ allows us to apply the level set algorithm introduced in [11]. Unlike most level set methods, this algorithm is based on topological sensitivity information rather than on shape sensitivities.

We represent a design by means of a level set function ψ in the following way: Wherever ψ is positive, we have ferromagnetic material, and wherever it attains negative values, we have air, i.e.,

$$\psi(x) > 0 \quad \Leftrightarrow \quad x \in \Omega_f,$$
$$\psi(x) < 0 \quad \Leftrightarrow \quad x \in \Omega_{\mathrm{air}}.$$

By this definition, the interface between Ω_f and Ω_{air} is given by the zero level set of the function ψ. The algorithm is based on the following observation: Defining the generalized topological derivative,

$$\tilde{G}(x) = \begin{cases} G^{f\to\mathrm{air}}(x) & x \in \Omega_f, \\ -G^{\mathrm{air}\to f}(x) & x \in \Omega_{\mathrm{air}}, \end{cases}$$

the condition $\psi = \tilde{G}_\psi$ is a sufficient optimality condition. Here, $\tilde{G}_\psi$ denotes the generalized topological derivative for the design represented by the level set function ψ. The optimality can be seen as follows: For a point x_0 in Ω_f, we have that $0 < \psi(x_0) = \tilde{G}_\psi(x_0) = G^{f\to air}(x_0)$. Thus, introducing air at point x_0 would not decrease the objective function. An analogous argument holds for a point x_0 in Ω_{air}. In each iteration of the algorithm, the next design is chosen as a combination of the current design and the generalized topological derivative in such a way, that a decrease of the objective function is achieved. The algorithm is the following:

1) Initialization: Choose ψ_0 with $\|\psi_0\| = 1$, compute $\tilde{G}_{\psi_0}$ and set $k = 0$
2) set the angle $\theta_k = \arccos\left(\psi_k, \|\tilde{G}_{\psi_k}\|^{-1}\tilde{G}_{\psi_k}\right)$ and update
$$\psi_{k+1} = \frac{1}{\sin\theta_k}\ [\sin((1-\kappa_k)\theta_k)\psi_k + \sin(\kappa_k\,\theta_k)\,\|\tilde{G}_{\psi_k}\|^{-1}\tilde{G}_{\psi_k}\,]$$
where $\kappa_k = \max\{1, 1/2, 1/4, \dots\}$ such that
$$J(\psi_{k+1}) < J(\psi_k)$$
3) Compute $\tilde{G}_{\psi_{k+1}}$
4) If $\tilde{G}_{\psi_{k+1}} = \psi_{k+1}$ then stop, else $k \leftarrow k+1$ and go to 2)

For details on the algorithm and a mathematical analysis, we refer the reader to [11] and [14]. We applied the algorithm to the optimization of the electric motor described in Section II. The initial and final design are depicted in Fig. 3. The objective functional J was reduced by 92%, from $J_{init} = 0.4550$ to $J_{final} = 0.0362$. The THD was decreased from 0.1350 to 0.0111 and the amplitude A_1 of the first harmonic was increased from 0.2968 to 0.3067. We want to emphasize that this algorithm is very robust with respect to the choice of the initial design and can achieve large topological changes.

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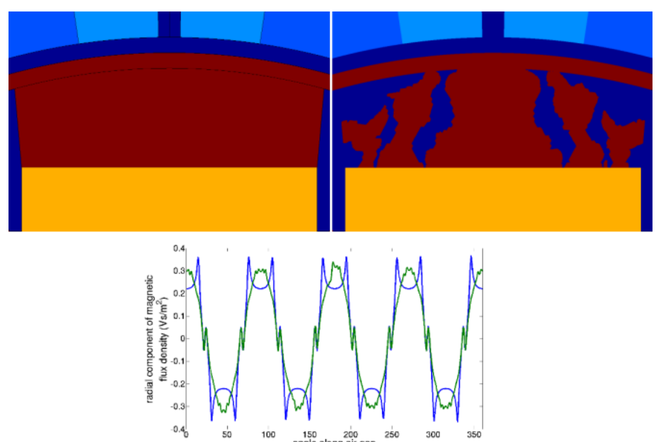

Fig. 3. Results obtained by application of level set algorithm: initial design (top left), final design (top right). Bottom: radial component of magnetic flux density along air gap for initial design (blue curve) and final design (green curve).

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